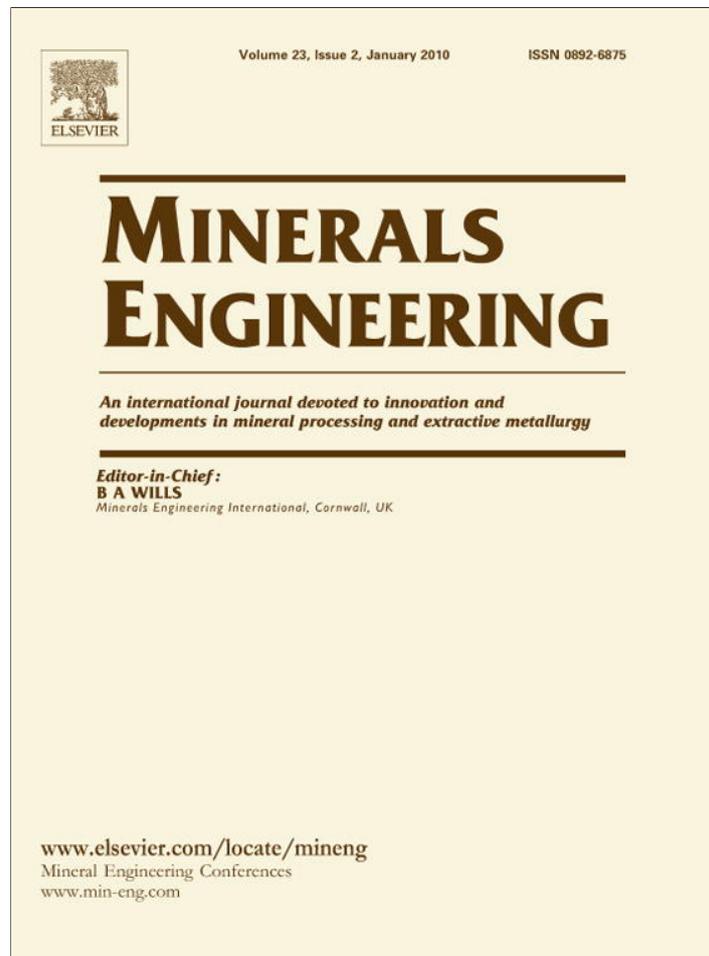


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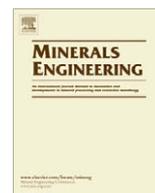
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## Geometallurgical modelling of the Collahuasi flotation circuit

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## ABSTRACT

The flotation rate constant was modelled as a function of air dispersion properties and the complete feed particle size distribution by using the collision–attachment–detachment approach and introducing a parameter ( $\phi$ ) which represents the inherent floatability of the ore. It was found that this parameter  $\phi$  is characteristic of the geometallurgical unit and does not depend on the main operating conditions. The parameter  $\phi$  is dimensionless and can be estimated either from laboratory testwork or directly from an industrial kinetics survey and can be used to predict industrial operation, provided that the other components of the model are evaluated under actual operation conditions. An empirical expression for the maximum achievable recovery – infinite time recovery – is also presented. The complete model, including flotation rate constant and infinite time recovery, was tested showing good correlation at both laboratory and industrial scale. At industrial scale the model was able to predict metallurgical results in a time frame of several weeks at Compañía Minera Doña Inés de Collahuasi SCM, showing an average relative error of less than 2%.

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## 1. Introduction

Compañía Minera Doña Inés de Collahuasi SCM initiated the development of a new geometallurgical model to characterize its Rosario deposit in terms of its comminution circuit capacity (Alruiz et al., 2009) and flotation performance. The flotation component of the model is now described in detail.

The flotation process is affected, among other factors, by the dispersion of bubbles and particles. The dispersion of gas into bubbles – gas dispersion – has a key impact on the flotation performance, as described in the literature (Schwartz and Alexander, 2006). Gas dispersion in the collection zone can be characterized by several variables, namely, the superficial gas velocity ( $J_g$ ), the gas holdup ( $\varepsilon_g$ ), the Sauter mean bubble size ( $d_b$ ), and the bubble surface area flux ( $S_b = 6J_g/d_b$ ) (Nesset et al., 2006). The latter being perhaps the most important gas dispersion property since it represents the flux of available bubble surface for flotation (Gorain et al., 1997, 1998).

Scaling-up laboratory results to industrial scale is a fundamental problem both for designing new operations and for the optimization of existing ones. The similitude approach is usually applied, where similarities between laboratory and industrial conditions are established by using a series of dimensionless parameters such as the Power number, Flux number, and Reynolds number. Another commonly used approach refers to the use of purely empirical cor-

relations (Vallebuona et al., 2003) where regression techniques and statistical analysis of experimental data from laboratory tests are used. Other authors have modelled the flotation rate parameter as a function of operational parameters such as agitation level (rpm), gas flow rate and particle size (Gorain et al., 1997, 1998, 1999; Kracht et al., 2005).

In the current approach, the flotation rate constant for two Collahuasi's geometallurgical units (UGM, from the Spanish "Unidad GeoMetalurgica") are modelled as a function of the gas dispersion properties and the flotation feed particle size distribution, along with operational and equipment parameters. The approach has the advantage of allowing the simulation of flotation performance at different scales. The model introduces a dimensionless parameter ( $\Phi$ ), characteristic of the ore type, that can be used to scale-up results from the laboratory to industrial scale.

## 2. Flotation kinetics modelling in conventional cells

Several authors have modelled flotation as a first order process (Kelsall, 1961; Arbiter and Harris, 1962; Mao and Yoon, 1997), characterized by a flotation rate constant  $k$ :

$$\frac{dN_p}{dt} = -kN_p \quad (1)$$

where  $N_p$  is the corresponds to the particle concentration in the machine.

Eq. (1) may be written as follows (Sherrell, 2004):

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$$\frac{dN_p}{dt} = -kN_p = -Z_{pb}P_A(1 - P_D)R_f \quad (2)$$

where  $Z_{pb}$  is the collision frequency between particles (index  $p$ ) and bubbles (index  $b$ ),  $P_A$  is the probability of attachment,  $P_D$  the probability of detachment and  $R_f$ , the froth recovery, which, in the case of low froth depth, tends towards 1 (Gorain et al., 1998). For the purpose of this article, the froth recovery will be assumed to be unity.

From Eq. (2), the flotation rate constant can be written as:

$$k = \frac{Z_{pb}}{N_p} P_A(1 - P_D)R_f \quad (3)$$

### 2.1. Attachment and detachment probability

For flotation modelling, the attachment and detachment processes can be estimated using Eqs. (4) and (5) (Yoon and Luttrell, 1989).

The attachment probability can be estimated as follows:

$$P_A = \text{sen}^2 \left[ 2 \cdot \arctan \left( \exp \left( \frac{-(45 + 8 \cdot \text{Re}_b \cdot t_i \cdot U_b)}{15 \cdot d_b \cdot \left(\frac{d_p}{d_b} + 1\right)} \right) \right) \right] \quad (4)$$

where  $\text{Re}_b$  is the bubble Reynolds number.

The detachment probability,  $P_D$ , is given by:

$$P_D = \left( \frac{d_p}{d_{p,\text{max}}} \right)^{1.5} \quad (5)$$

where  $d_{p,\text{max}}$  is the maximum floatable particle size fed into the machine, which for this article will be defined as  $P_{95}$ , i.e., the particle size under which 95% of the particles can be found.

Eqs. (4) and (5) are empirical formulae, valid for single particle size and do not reflect the behaviour of complete particle size distribution. Therefore, in order to calculate the flotation probabilities described above, two expressions representing weighted average flotation probabilities are introduced:

$$P_A = \sum_{i=1}^n P_A^i \cdot f_i, \quad P_D = \sum_{i=1}^n P_D^i \cdot f_i \quad (6)$$

where  $f_i$  represents the retained fraction in the particle size distribution, and  $n$  is the number of fractions in the particle size distribution. This weighted average flotation probabilities allows estimating the effect of the feed particle size distribution on flotation kinetics.

### 2.2. Bubble size estimation

The bubble size can be estimated using Eq. (7), which is an adaptation (Vallebuona et al., 2003) of Gorain's expression (Gorain et al., 1999):

$$d_b = \frac{6}{a \cdot N_s^b \cdot J_g^{c-1} \cdot A_s^d \cdot P_{80}^e} \quad (7)$$

where  $N_s$  is the peripheral impeller velocity in (m/s);  $J_g$  is the superficial gas velocity (cm/s) and  $A_s$  is the impeller aspect ratio.

Data from survey campaigns with direct measurements of  $J_g$  and  $d_b$  are necessary to estimate the specific values for the parameters  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  (Gorain et al., 1999; Vallebuona et al., 2003). Although Eq. (7) does not consider frother concentration as a parameter, it works well in cases where the frother dosage is close or beyond the critical coalescence concentration, which is a common practice at industrial scale (Laskowski, 2003).

The model presented in this article considers the Sauter mean bubble size as a single bubble diameter representing the complete bubble size distribution. Further development of this model to in-

clude the complete bubble size distribution is under way and will be presented in a future article.

### 2.3. Collision frequency in turbulent flow

According to Abrahamson's model (Abrahamson, 1975) for turbulent flow,  $Z_{pb}$  may be written as follows:

$$Z_{pb} = 2^{2/3} \pi^{1/2} N_p N_b \left( \frac{d_p + d_b}{2} \right)^2 \sqrt{\bar{U}_p^2 + \bar{U}_b^2} \quad (8)$$

where  $N_p$  is the particle concentration expressed as number density and  $N_b$  is the number density of bubbles.  $\bar{U}_p$  is the particle velocity (Liepe and Moeckel, 1976) and  $\bar{U}_b$  is the bubble velocity (Lee and Erickson, 1987), which can be expressed as follows:

$$\sqrt{\bar{U}_p^2} = 0.4 \frac{\varepsilon^{4/9} d_p^{7/9}}{\nu^{1/3}} \left( \frac{\rho_s - \rho_l}{\rho_l} \right)^{2/3} \quad (9)$$

$$\bar{U}_b^2 = C_o (\zeta d_b)^{2/3} \quad (10)$$

$C_o$  is given by Batchelor (1951) as 2 and  $\zeta$  corresponds to the energy dissipation rate.

The number density of bubbles ( $N_b$ ) can be written as a function of gas holdup ( $\varepsilon_g$ ) and bubble size ( $d_b$ ):

$$N_b = \frac{3\varepsilon_g}{4\pi d_b^3} \quad (11)$$

The gas holdup, on the other hand, can be estimated from the  $S_b$  as follows (Finch et al., 2000):

$$\varepsilon_g = \frac{S_b}{5.5} \quad (12)$$

Since none of the expressions used in the model accounts for the inherent floatability of the ore, a dimensionless parameter  $\Phi$  is included. Therefore, the kinetic rate constant, for turbulent flow conditions, can be expressed as follows:

$$k = \Phi \frac{3\varepsilon_g}{(2\pi)^{1/2} d_p^3} \left( \frac{d_p + d_b}{2} \right)^2 \sqrt{\bar{U}_p^2 + \bar{U}_b^2} P_A(1 - P_D)R_f \quad (13)$$

For simplicity, Eq. (13) may be written as:

$$k = \Phi \cdot F \quad (14)$$

where  $F$  corresponds to:

$$F = \frac{3\varepsilon_g}{(2\pi)^{1/2} d_p^3} \left( \frac{d_p + d_b}{2} \right)^2 \sqrt{\bar{U}_p^2 + \bar{U}_b^2} P_A(1 - P_D)R_f \quad (15)$$

Note that  $F$ , as expressed in Eq. (15), has units of 1/s and has to be converted to 1/min in order to be used in Eq. (14).

The parameter  $\Phi$  should not be confused with the parameter  $P$  in the  $k - S_b$  model proposed by Gorain et al. (1998). In that model,  $P$  accounts for the floatability of the ore, and implicitly for everything that  $S_b$  does not account for, i.e., probability of attachment, detachment and collision frequency. In the current model, since all those variables are explicitly included in  $F$ , the parameter  $\Phi$  accounts only for the inherent floatability of the ore.

### 2.4. Scaling-up from laboratory to industrial scale

$F$  takes different values depending on the operating variables, which may differ between laboratory and industrial scale. Therefore, the expression depends on parameters such as impeller size and aspect ratio, together with gas dispersion properties (bubble size and gas holdup). On the other hand,  $\Phi$  accounts for the inherent floatability of the particles in the collection zone, which means it is unique for a given geometallurgical unit and can be deter-

**Table 1**  
Geometallurgical units defined for Rosario deposit within the period 2008–2012.

UGM	Alteration	Lithology	Proportion by mass (%)
1	Sericite; argillic; chlorite–sericite	Intrusive porphyry	18
2	Sericite; argillic; chlorite–sericite	Hosted rock	26
3	Quartz–sericite; porphyritic; quartz; biotite; potassic	Intrusive porphyry	19
4	Quartz–sericite; porphyritic; quartz; biotite; potassic	Hosted rock	25
5	Sericite; argillic; chlorite–sericite	Intrusive porphyry + hosted rock	7
6	Quartz–sericite; porphyritic; quartz; biotite; potassic	Intrusive porphyry + hosted rock	5

**Table 2**  
Mineralogical composition of UGM1 and UGM5.

Mineral	% Weight	
	UGM1	UGM5
Chalcopyrite	2.10	3.00
Chalcocite	0.18	0.41
Coveline	tr	tr
Enargite	tr	0.16
Bornite	0.48	tr
Soluble Cu	0.07	tr
Cuprite	tr	tr
Sphalerite	tr	tr
Molybdenite	0.15	0.17
Rutile	0.26	0.15
Pyrite	2.39	5.10
Hematite	tr	0.19
Magnetite	0.16	0.37
Limonite	tr	tr
Gangue	94.23	90.46
Total	100	100

**Table 3**  
Operating conditions for laboratory experimental campaign.

Test	$P_{80}$ , $\mu\text{m}$	Agitation, rpm	$J_g$ , cm/s	$N_s$ , m/s	$d_b$ , mm
1	200	1324	0.97	5.9	2.3
2	250	1324	0.97	5.9	2.6
3	250	1000	0.97	4.5	2.9
4	250	700	0.97	3.1	3.4
5	250	1324	0.70	5.9	2.4
6	250	1324	0.43	5.9	2.1
7	230	1324	0.97	5.9	2.5
8	300	1324	0.97	5.9	2.8
9	560	1324	0.97	5.9	3.6
10	250	1324	0.97	5.9	2.6

Note: Air flow rates reported at 20 psi and 20 °C. Bubble size estimated from Eq. (7).

mined at laboratory scale and used to forecast the flotation kinetics at industrial scale, provided that the system chemistry remains the same (pH, reagent types and dosages). Eq. (14) can be rewritten to yield  $K^{\text{INDUSTRIAL}}$ , the flotation rate constant at industrial scale, as follows:

$$K^{\text{INDUSTRIAL}} = \Phi \cdot F^{\text{INDUSTRIAL}} \quad (16)$$

$\Phi$  in Eq. (16) can be determined at the laboratory, whereas  $F^{\text{INDUSTRIAL}}$  is obtained from evaluating Eq. (15) at the industrial operating conditions.

### 3. Results and discussion

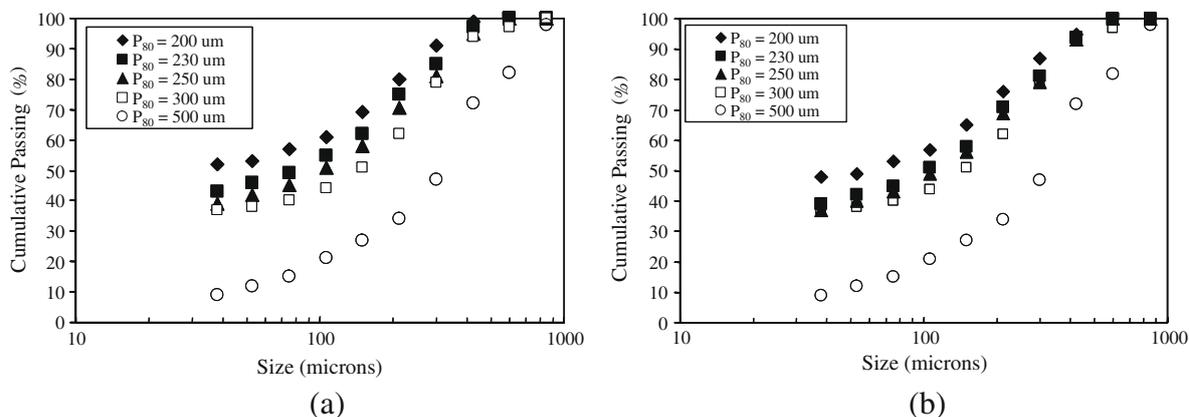
Collahuasi's Rosario deposit is characterized by six geometallurgical units (UGM1–UGM6). The definition of the geometallurgical units was made in two stages, based on previous work developed by the Xstrata Process Support Group, Canada (Fragomeni et al., 2005; Lotter et al., 2002). These stages involved firstly grouping the geometallurgical units on the basis of geological similarities and secondly determining their volumetric fraction in the material that was planned to be processed over the period 2008–2012.

The characteristics of each unit (alteration and lithology), along with the proportion in the deposit, are presented in Table 1.

#### 3.1. Flotation rate constant at laboratory scale

A laboratory flotation test program was carried out using samples of two different geometallurgical units (UGM) from Doña Inés de Collahuasi Mining Company, UGM1 and UGM5, which represent 18% and 7% of the total ore to be processed during the following 3 years, respectively. The mineral was analyzed by microscopy and QEMSCAN, and the composition of both geometallurgical units is presented in Table 2. The average copper grades are 1.12% for UGM1 and 1.37% for UGM5.

The experimental test program carried out with each geometallurgical unit consisted of a series of flotation tests in a Denver lab-



**Fig. 1.** Particle size distributions at different grinding levels: (a) UGM1 and (b) UGM5.

**Table 4**  
Laboratory rougher kinetics experimental campaign, UGM1.

Test	$\Phi_{UGM-1} = 77 \times 10^{-6}$		
	Klimpel parameters		$k$ Modelled
	$K_{Klimpel}$ , $\text{min}^{-1}$	$R_{scr}$ , %	$k_{Model}$ , $\text{min}^{-1}$
1	4.9	93.9	5.7
2	4.5	89.1	4.1
3	3.2	86.5	2.6
4	1.1	87.0	1.4
5	4.6	89.6	3.7
6	4.0	89.3	2.8
7	4.5	93.1	4.8
8	4.0	83.7	3.4
9	1.8	60.4	1.3
10	4.4	87.4	4.3

**Table 5**  
Laboratory Rougher kinetics experimental campaign, UGM5.

Test	$\Phi_{UGM-5} = 10.2 \times 10^{-5}$		
	Klimpel parameters		$k$ Modelled
	$K_{Klimpel}$ , $\text{min}^{-1}$	$R_{scr}$ , %	$k_{Model}$ , $\text{min}^{-1}$
1	6.6	95.1	7.3
2	5.3	91.8	5.5
3	2.5	88.3	3.3
4	2.1	87.3	1.7
5	5.1	91.5	4.7
6	4.8	92.0	3.7
7	6.3	93.3	6.1
8	4.9	89.5	4.4
9	2.6	78.1	1.7
10	4.3	91.7	5.3

oratory flotation cell modified to receive forced air from an air injection system. The studied variables were the grinding size ( $P_{80}$ ), agitation (rpm), and gas flow rate ( $J_g$ ). Ten different combinations of these variables were tested for each geometallurgical unit (see Table 3). The flotation tests were done under conditions as used in Collahuasi concentrator. The collectors used were isopropyl ethyl thionocarbamate and mercaptobenzothiazol (Matcol-123, 25 g/tons; Matcol-154, 19 g/tons), the frothers used were a mix of pine oil, alcohols, ethers and aldehydes (Matfroth-421, 35 g/

**Table 6**  
Parameters of Eq. (17) for UGM1 and UGM5.

Parameter	UGM 1	UGM 5
$\lambda_1$	0.083	0.028
$\lambda_2$	2.96	4.45
$\lambda_3$	116.58	109.95
$R^2$	0.99	0.96

tons; AX-133, 3 g/tons). Diesel was also added (15 g/tons) and the pH was adjusted to 9.5 with lime. The solids percentage in the tests was 30%.

Each  $P_{80}$  in Table 3 corresponds to a different particle size distribution (see Fig. 1) and was achieved by varying the grinding time of the ore sample in the laboratory. Especial attention was paid to the grinding process in the laboratory in order to obtain particle size distributions that represented those generated at industrial scale.

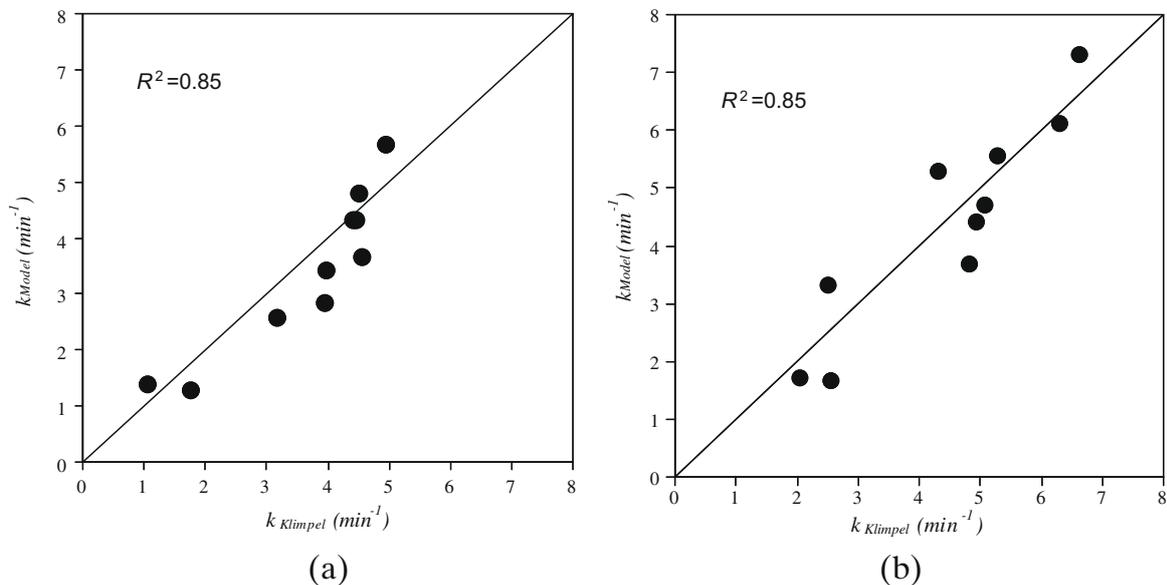
For each test, a first order kinetics (Klimpel, 1980) was fitted in order to obtain the experimental flotation rate constant  $k_{Klimpel}$ . The flotation rate constant for each test was also predicted using the expression proposed in Eq. (13). The value of  $F$  (Eq. (15)) was calculated for every test and  $\Phi$  was determined for both geometallurgical units by minimizing the sum of squared errors between experimental and modelled (predicted) flotation rate constant.

Tables 4 and 5 summarize the experimental results obtained for each geometallurgical unit together with the estimated (modelled) values of the flotation rate constant. The value of  $\Phi$  is also indicated.

The quality of the prediction can be observed in Fig. 2. The correlation coefficient between the experimental values (the Klimpel fitted flotation rate parameters) and the modelled flotation rate parameters was  $R^2 = 0.85$  for both geometallurgical units tested.

3.2. Infinite time recovery

From a statistical analysis of the results, it was found that infinite time recovery is strongly influenced by the bubble size ( $d_b$ ) and particle size ( $P_{80}$ ), which allowed the generation of an expression that relates these variables (see Eq. (17)).



**Fig. 2.** Experimental rate  $k_{Klimpel}$  versus modelled  $k$  for UGM1 (a) and UGM5 (b).

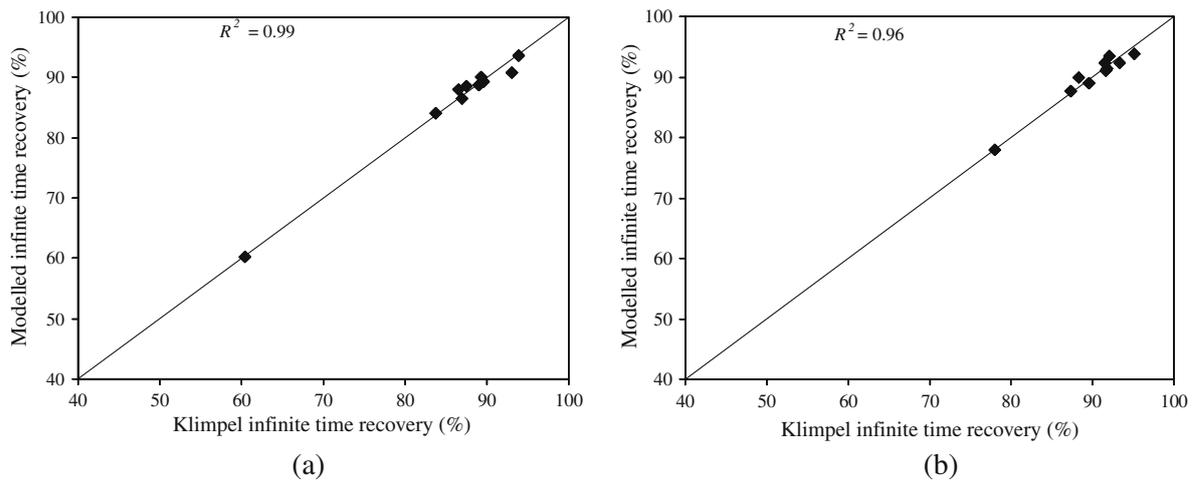


Fig. 3. Infinite time recovery  $R_{\infty}$  for UGM1 (a) and UGM5 (b).

Table 7

$\phi$  Values for each geometallurgical unit.

UGM	$\phi \times 10^{-5}$
1	7.7
2	8.3
3	13.2
4	7.2
5	10.2
6	11.7

$$R_{\infty} = -\lambda_1 \cdot P_{80} - \lambda_2 \cdot d_b + \lambda_3 \quad (17)$$

The parameters  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  in Eq. (17) depend on the ore type (UGM) and system chemistry. Table 6 shows the parameters obtained for each UGM along with the correlation coefficients ( $R^2$ ). Fig. 3 shows the quality of the fit.

According to Eq. (17), the bubble size ( $d_b$ ) can be influenced by parameters such as  $P_{80}$ , air flow rate, impeller speed and aspect ra-

tio. The same would apply to  $R_{\infty}$ , which means that the infinite time recovery depends on the grinding size ( $P_{80}$ ), as expected, but also depends on the main operating conditions and the impeller design.

### 3.3. Predicting flotation performance at industrial scale

Each UGM corresponds to a different mineral type, and hence, it has a characteristic  $\phi$  value. In order to determine the values of  $\phi$  for each geometallurgical unit, the same laboratory work described for UGM1 and UGM5 was repeated. The  $\phi$  values are presented in Table 7.

From January 2008 to April 2009, weekly data was collected from the production records. Information included throughput, the  $P_{80}$  per grinding line, copper head grade and the particle size distribution fed to the flotation circuit. For each geometallurgical unit, the industrial flotation rate constant was estimated using the Eq. (16). The average agitation level, air flow rate and impeller

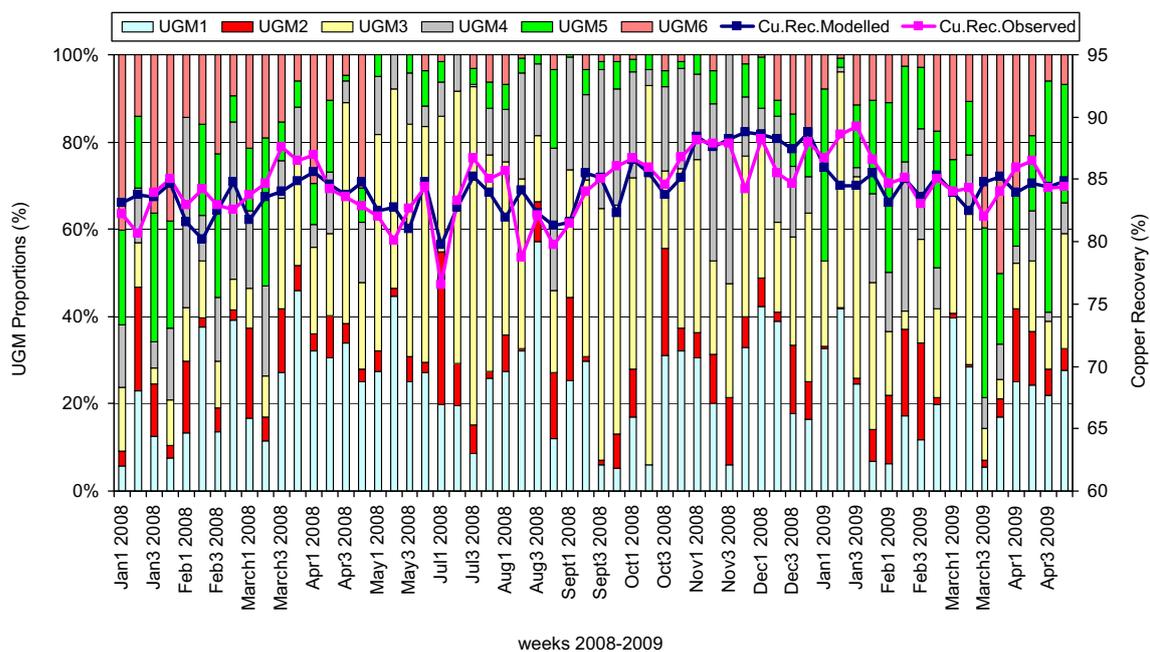


Fig. 4. Comparison between observed and modelled copper recoveries.

**Table 8**

Prediction of copper recovery using  $\phi$  value from industrial survey on Day 1.  $\phi = 7.8 \times 10^{-5}$ .

Day	Throughput, tpd	Cu feed grades, %	Cu recovery, %	
			Observed	Modelled
1	123,000	1.10	83.5	83.3
2	131,160	1.21	83.3	83.2
3	121,080	1.12	82.7	82.7
4	122,448	1.19	84.6	84.1
5	125,832	1.15	85.3	85.0
6	126,576	1.17	82.6	82.6

sizes are known for Collahuasi's flotation circuit and for each period or week. The equations presented above were incorporated into a flotation simulator software programmed in Visual C++ in order to evaluate the performance of the flotation circuit for different operating conditions.

The proportions of each UGM fed to the plant are known from the mining planning report for the period, which allows estimating the flotation performance with the model presented in this article and comparing it with the actual flotation performance in the plant (from production records). Fig. 4 shows the predictive capacity of the model from January 2008 to April 2009. The proportion of each metallurgical unit fed to the plant is shown for each period.

As it can be seen, the flotation model was able to predict in a satisfactory manner the observed copper recoveries. For each week, the copper recovery for the blend of geometallurgical units was calculated estimating the individual copper recovery for each UGM and then calculating the weighted average using the mass fraction of each UGM in the blend. The calculated average relative error of the model was 1.8%. Note that the good fit of the model supports the assumption of froth recovery equal to 1.

Since each geometallurgical unit has a characteristic  $\phi$ , and the model allows scaling-up results between laboratory and industrial scales, the value of  $\phi$  could also be determined directly from an industrial kinetic survey and used to model the flotation performance for other days where the same ore (or blend of UGM) is being processed. This was tested at Doña Inés de Collahuasi Mining Company concentrator and the results are presented in Table 8.

After an initial survey performed at the plant (Day 1 in Table 8) to determine the parameter  $\phi$  corresponding to the ore being processed, the model was able to predict the recovery of several days of operation at the Doña Inés de Collahuasi Mining Company concentrator. This indicates that the parameter  $\phi$  can be obtained either from laboratory tests or from kinetic survey in the actual plant.

#### 4. Conclusions

The results presented in this article show that it is possible to estimate flotation performance in terms of the operating conditions and the complete particle size distribution of the flotation feed. The model considers the probability of flotation and includes a parameter  $\phi$  that accounts for the inherent floatability of the mineral.

The parameter  $\phi$  is a key component in the model, it is characteristic of the ore type or geometallurgical unit and it does not depend on the scale of the flotation process. Once  $\phi$  is determined, it is possible to scale-up the kinetics parameter in order to estimate copper recoveries at industrial scale. This approach was tested with data collected from the production records at Collahuasi's

concentrator (from January 2008 to April 2009). The results showed a good prediction of the actual recovery, with an average relative error of 1.8%.

The infinite time recovery  $R_{\infty}$  was empirically modelled as a function of  $P_{80}$  and the main operating conditions. The correlation coefficient between experimental and modelled values was over 0.95 for the geometallurgical units studied.

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